Modeling Dividends and Other Distributions
A Case Study

Gary Schurman, MBE, CFA

December, 2020

In this white paper we will model the effect on future asset value of (1) paying no dividends, (2) paying excess cash flow out as dividends, and (3) depositing excess cash flow into a company-owned bank account. To that end we will use the following hypothetical problem...

Our Hypothetical Problem

Our valuation options are...

Table 1: Valuation Options

Option	Description
1	Assume dividends are zero
2	Assume dividends are equal to excess cash flow
3	Assume dividends are not paid out but rather are reinvested in cash-equivalents

We are given the following go-forward model assumptions...

Table 2: Go-Forward Model Assumptions

Description	Value
Annalized cash flow at time zero ($)	1,000,000
Annualized cash flow growth rate (%)	5.00
Annualized cash flow growth rate volatility (%)	35.00
Risk-adjusted discount rate (%)	20.00
Risk-free interest rate (%)	3.00

Note that the rates quoted in the table above are discrete-time rates. Our task is to answer the following questions given that the random draw from a normal distribution with mean zero and variance one was [0.50]...

Question 1: What is asset value at the end of year 3 given Valuation Option 1 in the table above?

Question 2: What is asset value at the end of year 3 given Valuation Option 2 in the table above?

Question 3: What is asset value at the end of year 3 given Valuation Option 3 in the table above?

Asset Value at Time Zero

We will define the variable C_t to be random annualized cash flow at time t, the variable λ to be the continuous-time expected cash flow growth rate, and the variable σ to be the cash flow growth rate volatility. The equation for random annualized cash flow at time t is...

$$C_t = C_0 \,\mathrm{Exp}\left\{\left(\lambda - \frac{1}{2}\sigma^2\right)t + \sigma\sqrt{t}\,Z\right\} \text{ ...where... } Z \sim N\Big[0,1\Big] \tag{1}$$

Noting that annualized cash flow in Equation (1) above is a lognormally-distributed random variable the equation for expected annualized cash flow at time t is...

$$\mathbb{E}\Big[C_t\Big] = C_0 \,\mathrm{Exp}\Big\{\lambda\, t\Big\} \text{ ...where... } \lambda = \ln\Big(1 + \text{discrete-time cash flow growth rate}\Big) \tag{2}$$

We will define the variable A_0 to be asset value at time zero and the variable κ to be the continuous-time risk-adjusted discount rate. Using Equation (2) above the equation for asset value at time t is...

$$A_0 = \int\limits_0^\infty \mathbb{E}\bigg[C_u\bigg] \, \mathrm{Exp}\bigg\{-\kappa\, t\bigg\}\,\delta u \;\; \text{...where...} \;\; \kappa = \ln\bigg(1 + \text{discrete-time discount rate}\bigg) \tag{3}$$

Using Equation (2) above the solution to valuation Equation (3) above is... [1]

$$A_0 = \int\limits_0^\infty C_0 \, \mathrm{Exp}\bigg\{\bigg(\lambda - \kappa\bigg)t\bigg\}\,\delta u = C_0\bigg(\kappa - \lambda\bigg)^{-1} \tag{4}$$

Using Table 2 above we will make the following variable definitions...

$$C_0 = 1{,}000{,}000 \;\; \text{...and...} \;\; \lambda = \ln\bigg(1 + 0.05\bigg) = 0.0488 \;\; \text{...and...} \;\; \kappa = \ln\bigg(1 + 0.20\bigg) = 0.1823 \tag{5}$$

Using Equations (4) and (5) above the value of our hypothetical company at time zero is...

$$A_0 = 1{,}000{,}000 \times \bigg(0.1823 - 0.0488\bigg)^{-1} = 7{,}490{,}600 \tag{6}$$

Modeling Asset Value Over Time

We will define the variable μ to be the continuous-time expected asset total rate of return, the variable ϕ to be the continuous-time expected asset dividend yield, and the variable σ to be the expected cash flow growth rate volatility. Using Equation (5) above and Table 2 above we will make the following variable definitions... [2]

$$\mu = \kappa = 0.1823 \;\; \text{...and...} \;\; \phi = \mu - \lambda = 0.1335 \;\; \text{...and...} \;\; \sigma = 0.3500 \tag{7}$$

Using Equation (7) above we will define the normally-distributed random variable θ to be the following equation... [2]

$$\theta = \bigg[\bigg(\mu - \phi - \frac{1}{2}\sigma^2\bigg)t + \sigma\sqrt{t}\,Z\bigg]\bigg/ t \;\; \text{...where...} \;\; Z \sim N\bigg[0, 1\bigg] \tag{8}$$

Using Equation (8) above the equation for random asset value at time t is...

$$A_t = A_0 \, \mathrm{Exp}\bigg\{\theta\, t\bigg\} \;\; \text{...where...} \;\; \theta \sim N\bigg[\bigg(\mu - \phi - \frac{1}{2}\sigma^2\bigg)t,\, \sigma^2 t\bigg] \tag{9}$$

We will define the variable B_t to be the random bank account value at time t and the variable α to be the continuous-time risk-free rate. The bank account is used to accumulate dividends and other distributions retained by the company. The bank account is assumed to be invested at the risk-free rate. Using Equations (6), (7), and (8) above the equation for the bank account is... [2]

$$B_t = \frac{\phi}{\theta - \alpha} A_0 \, \mathrm{Exp}\bigg\{\alpha\, t\bigg\}\bigg(\mathrm{Exp}\bigg\{(\theta - \alpha)\, t\bigg\} - 1\bigg) \tag{10}$$

Using Table 2 above we will make the following variable definition...

$$\alpha = \ln\bigg(1 + 0.03\bigg) = 0.0296 \tag{11}$$

The Answers To Our Hypothetical Problem

Question 1: What is asset value at the end of year 3 given Valuation Option 1 in the table above?

Using Equations (7) and (8) above the value of random variable θ is...

$$\theta = \bigg[\bigg(0.1823 - 0 - \frac{1}{2} \times 0.3500^2\bigg) \times 3 + 0.3500 \times \sqrt{3} \times 0.50\bigg]\bigg/ 3 = 0.2221 \tag{12}$$

Using Equations (9) and (12) above the answer to the question is...

$$A_3 = 1,000,000 \times \text{Exp}\left\{0.2221 \times 3\right\} = 1,947,000 \tag{13}$$

Question 2: What is asset value at the end of year 3 given Valuation Option 2 in the table above?

Using Equations (7) and (8) above the value of random variable θ is...

$$\theta = \left[\left(0.1823 - 0.1335 - \frac{1}{2} \times 0.3500^2\right) \times 3 + 0.3500 \times \sqrt{3} \times 0.50\right] \Big/ 3 = 0.0886 \tag{14}$$

Using Equations (9) and (12) above the answer to the question is...

$$A_3 = 1,000,000 \times \text{Exp}\left\{0.0886 \times 3\right\} = 1,019,900 \tag{15}$$

Question 3: What is asset value at the end of year 3 given Valuation Option 3 in the table above?

Using Equation (10) above the cash account balance at the end of year 3 is...

$$B_3 = \frac{0.1335}{0.0886 - 0.0296} \times 1,000,000 \times \text{Exp}\left\{0.0296 \times 3\right\} \times \left(\text{Exp}\left\{(0.0886 - 0.0296) \times 3\right\} - 1\right) = 478,900 \tag{16}$$

Using Equations (15) and (16) above the answer to the quesion is...

$$A_3 = 1,019,900 + 478,900 = 1,498,800 \tag{17}$$

References

[1] Gary Schurman, *Valuation in Discrete and Continuous,* September, 2017.

[2] Gary Schurman, *Modeling Dividends and Other Distributions - Part II,* December, 2020.